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Analysis of Power Density Levels

For

Raytheon Prototype Radar Demonstration System (PRDS)

Raytheon Request for FCC Special Temporary Authorization (STA)

STA File Number 0038-EX-ST-2011

STA Confirmation Number EL507664

(01/19/2011)

Overview

- The purpose of this presentation is to document the analysis of the power density levels produced by the Raytheon Prototype Radar Demonstration System (PRDS) to ensure compliance with all applicable RF-related safety standards.
  - The following slides summarize the expected power density levels with respect to human exposure and aircraft susceptibility. Results show no hazard exists for aircraft. Controls for personnel will be established, as described herein.
  - Power density levels will be verified to be within safe limits (per 47 CFR 1.1310) for personnel at the initial turn-on of the RF equipment and at any time test setup changes are made that affect power density levels of the test or surrounding areas.
  - Electromagnetic energy exposure control measures will be documented in an RF safety control plan describing elements such as signage, procedures, personnel training, and RF survey measurements.

Objective of PRDS RF Emissions and Exposure Limits Analysis

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- Conduct analysis to determine if RF emission levels will exceed safety levels accessible to personnel and aircraft
  - Results will provide safe permissible distances for personnel and equipment that correspond with Maximum Permissible Exposure (MPE) limits.
  - MPE limits have been determined from the following documentation:
    - 47 CFR 1.1310. “Radiofrequency Radiation Exposure Limits”
    - IEEE C95.1-2005. “Standard for Safety Levels with Respect to Human Exposure to Radio Frequency Electromagnetic Fields”
    - AFMAN91-201. “Explosives Safety Standards”
    - Mil-Std-461F. “Requirements for the Control of Electromagnetic Interference, Characteristics of Subsystems and Equipment”
    - FAA Notification 8110.71. “Guidance for the Certification of Aircraft Operating in a High Intensity Radiation Field (HIRF) Environment”
    - 14 CFR Aeronautics and Space; 23.1308, 25.1317, 27.1317, 29.1317

- Conduct a power density calculation along the peak of the beam from the antenna face up to 4km above the antenna
  - Confirm legitimacy of the analysis by comparing results against theory
  - Determine minimum safe aircraft altitude (confirm no impact to aircraft)

Summary of Applicable RF Exposure/Susceptibility Limits

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- The following table summarizes the MPE and susceptibility limits used in this analysis. These limits are derived from the reference texts listed on the previous slide:

Requirement Description	Specification	Peak	Average
		mW/cm^2	mW/cm^2
Aircraft EMI/Susceptibility			
Military	MIL-STD-461F, Table VII	10.6	
Civilian	N8110.71, Table 1	6631.4	28.9
Civilian VFR Rotorcraft	N8110.71, Table 2	2387.3	23.9
Civilian	14 CFR 23.1308 14 CFR 25.1317 14 CFR 27.1317 14 CFR 29.1317	6631.4	28.9
Electro-Explosive Devices (in or on aircraft)	AFMAN 91-201, Table 9.1	10.0	
Human Exposure Limit			
C95.1 Controlled	IEEE C95.1-2005, Table 8		10.0
C95.1 Uncontrolled	IEEE C95.1-2005, Table 9		1.0
FCC, Controlled	47 CFR 1.1310		5.0
FCC, Uncontrolled	47 CFR 1.1310		1.0

Overview of PRDS (1)

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- The PRDS will be tested outdoors at two locations:
  - Raytheon Facility in Pelham, NH
  - Raytheon Facility in Sudbury, MA
- System performance parameters will be the same at both test sites:

Attribute	Value	Comments
Physical Aperture Area	$1m^2$ (1m x 1m)	
Peak Transmit Power	14.6kW (Maximum)	Includes front-end losses
Transmit Antenna Gain	40.1dBi	Includes element gain
Operating Frequency Range	9.3 - 10 GHz	Tunable in 1Hz increments
Waveform types	Pulsed Linear/Non-Linear FM (chirp) Pulsed Non-FM (pulsed CW/unmodulated) Pulsed Phase Coded (Bi-phase Barker)	
Waveform Instantaneous Bandwidth	10 MHz (Maximum)	
Pulse width	15 µs (Maximum)	
Rotation Rate	30 rpm (Maximum)	
Transmit Duty Cycle	10% (Maximum)	
Azimuth 3-dB Beamwidth	1.6°	
Elevation 3-dB Beamwidth	1.7°	
Antenna Height	8ft (2.4m) to center of antenna	Array is mounted on a portable trailer
Antenna Orientation	Tilted 10° back off horizon	

Overview of PRDS (2)

- The azimuth and elevation coverage will differ between the two test sites:

Attribute	Raytheon Facility (Pelham, NH)	Raytheon Facility (Sudbury, MA)
Azimuth Coverage*	+/- 90°	+/- 30°
Elevation Coverage	0° to 55° (with respect to Earth Horizontal)	7° to 55° (with respect to Earth Horizontal)

- The azimuth coverage stated here for each site describes the maximum coverage radiated by the antenna for a fixed antenna.
  - After initial integration & test, the antenna will be rotating at 30rpm. Software-defined sector blanking zones will prevent the antenna from emitting radiation outside of the defined azimuth coverage areas for each site.
  - When the antenna is rotating, the effective duty factor of the radar is degraded, so the average power density decreases. The minimum safe distances for the rotating antenna case will be less than when the antenna is fixed.

Power Density Calculation
Equation Used in Analysis

- The power density at any observation point from the array face is approximated by (see Addendum and References for derivation):

$$P_T(r,\theta,\phi) = \frac{P_{rad,n} DF}{4\pi} \left| \sum_{i=1}^{N} \sqrt{\cos^{\alpha}\theta_i}\, e^{-jk r_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i} \right|^2 \qquad (1)$$

*This equation is valid for both near and far-field distances

Where

$P_{rad,n} = \textit{Power Radiated Per Element}$

$\theta_i = \textit{Angle off boresight (relative to } i^{th} \textit{ element, rad)}$

$\alpha = \textit{Element Pattern Exponent}$

$N = \textit{Total number of elements in array}$

$k = \textit{Free-space wavenumber } (m^{-1})$

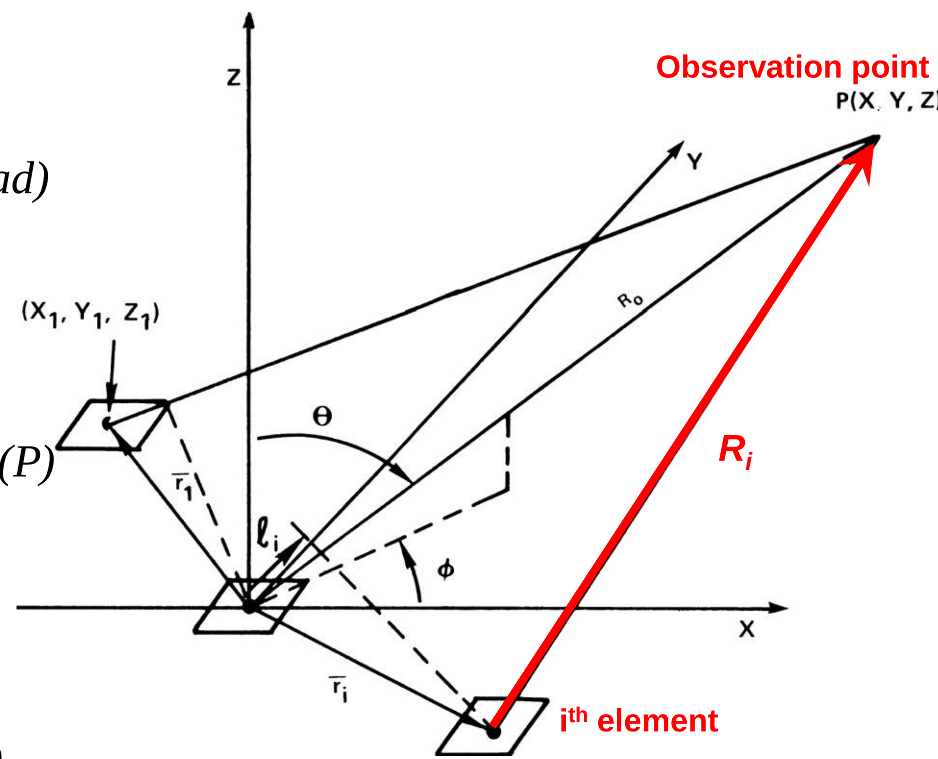
$R_n = \textit{Range from } i^{th} \textit{ element to observation point } (P)$

$= \sqrt{(x_{obs} - x_i)^2 + (y_{obs} - y_i)^2 + (z_{obs} - z_i)^2}$

$r_i = \textit{location of } i^{th} \textit{ element } (x_i, y_i, z_i)$

$\hat{r}_0 = \textit{pointing vector}$

$= \hat{x}u_0 + \hat{y}v_0 + \hat{z}\cos\theta_0 \textit{ (u,v are direction cosines)}$



Power Density Calculation
Theoretical Equations

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- The theoretical power density in the far-field can be expressed as:

$$Thoeretical\ Power\ Density = \frac{P_{TX} G_{TX} DF}{4\pi R^2} = (50.75 - 20\log_{10} R)\ {}^{dBm}\!/_{cm^2} \quad (2)$$

Where

$$P_{TX}\ (Total\ Peak\ Transmit\ Power) = 14600W$$

$$G_{TX}\ (Total\ Transmit\ Gain) = 40.1dBi$$

$$DF\ (Duty\ Factor) = 0.1$$

$$R = Range\ \ from\ array\ center\ to\ observation\ point$$

- Similarly, the power density at the Array face can be calculated as:

$$Power\ Density\ at\ Array\ Face = \left.\frac{P_{TX} DF}{Area_{Array}}\right|_{Area_{Array}=1m^2} = 21.64\ {}^{dBm}\!/_{cm^2} \quad (3)$$

- The far-field distance is calculated as:

$$R_{far-field} > \left.\frac{2D^2}{\lambda}\right|_{\substack{D=1m \\ \lambda=3\cdot10^8/9.6\cdot10^9=0.0313\,m}} \quad \therefore R_{far-field} > 63.9m \approx 64m \quad (4)$$

Power Density Calculation
Using Online Reference Tool

- Power Density Calculation along boresight calculated using "Amateur Radio RF Safety Calculator"
  - Same system parameters Input
  - (http://hintlink.com/power_density.htm)
  - This program uses the formulas given in the FCC OET Bulletin No. 65 to estimate power density in the main lobe of the antenna.
  - This program's calculation is limited because it cannot predict sidelobe radiation outside of the main lobe, and it cannot model the loss in gain as the main lobe is scanned off boresight.

Calculate Radio Frequency Power Density

The average power at the antenna:
In watts — 1460

The antenna gain in dBi:
Enter 2.2 for dipoles; add 2.2 for antennas rated in dBd — 40.1

The distance to the area of interest:
From the centre of the antenna, in feet — 400

The frequency of operation:
In MHz — 9600

Ground Reflection Effects

In most cases, the ground reflection factor is needed to provide a truly worst-case estimate of the compliance distance in the main beam of the antenna. Including the ground reflection effects may yield more accurate results especially with very low antennas, non-directional antennas, and calculations below the main lobe of directional antennas.

Do you wish to include effects of ground reflections? ○ Yes ◉ No

[Calculate RF Power Density] [Reset Values]

Calculation Results

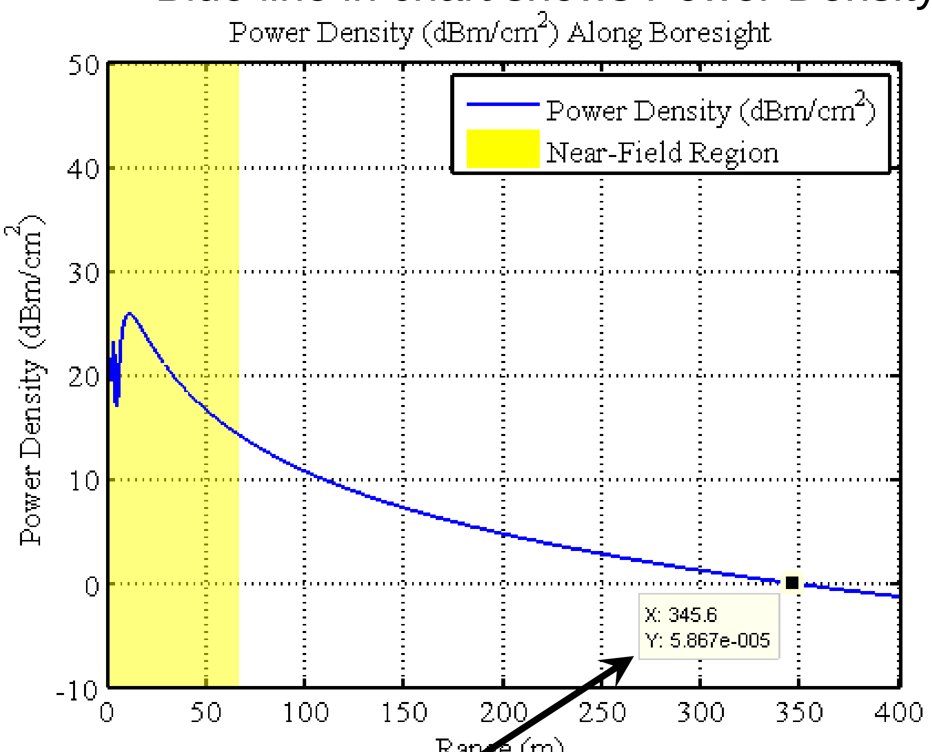
Average Power at the Antenna	1460 watts
Antenna Gain in dBi	40.1 dBi
Distance to the Area of Interest	400 feet 121.92 metres
Frequency of Operation	9600 MHz
Are Ground Reflections Calculated?	No
Estimated RF Power Density	7.9983 mW/cm^2

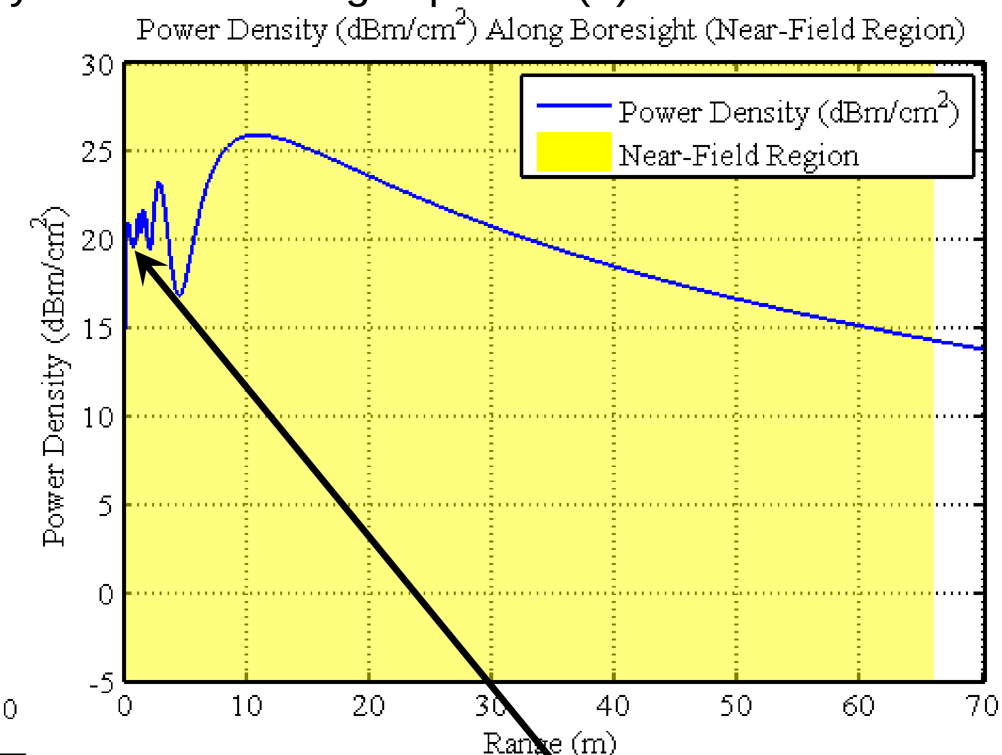
	Controlled Environment	Uncontrolled Environment
Maximum Permissible Exposure (MPE)	5.005 mW/cm^2	1.005 mW/cm^2
Distance to Compliance From Centre of Antenna	505.9582 feet 154.2161 metres	1131.2951 feet 344.8188 metres
Does the Area of Interest Appear to be in Compliance?	no	no

Power Density Calculation

Comparison Between Equations (1) – (3) and Online Reference Tool

- This shows a comparison between the Power Density calculation in equation (1) to equations (2), (3) and the online reference tool from page 9.
- Blue line in chart shows Power Density calculated using Equation (1)





Equation (1) shows minimum safe distance of **345.6m** along the main lobe. Equation (1) aligns closely with equation (2) and the online tool.

Equation (1) shows Power Density at 0m (array face) is **20dBm/cm²**. Equation (1) aligns closely with equation (3).

Near-Field and Far-Field Power Density Levels From Equation (1) Comply with Equations (2), (3) and Online Reference Tool

Power Density Calculation
Algorithm Description

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1.) Setup antenna source

2.) Select an observation point (or set of) from which to calculate the power density

– The range is taken from the i^{th} element in the array to some observation point in free-space.

3.) Calculate the power density at each observation point from equation (1)

4.) Compare range-dependent power densities to MPE limits defined

– Minimum safe distances assumed to be at ground level up to 8ft in height.

Power Density Calculations
Parameters and Assumptions

- The power density calculations that follow assume worst-case conditions for radiation. This includes:
  – Maximum Duty Cycle (10%)
  – Fixed (Non-Rotating) Antenna
  – The effect of ground reflectivity has been used to assume worst-case multipath conditions, per OET Bulletin 65 “Evaluating Compliance with FCC Guidelines for Human Exposure to Radiofrequency Electromagnetic Fields”.

Summary of Results to be Presented

- The following slides summarize the analysis results, Keep Out Zones and minimum safe distances for each test site.

Separate analysis for each site ensures accuracy and safety

Raytheon Facility in Sudbury, MA
Dimensions

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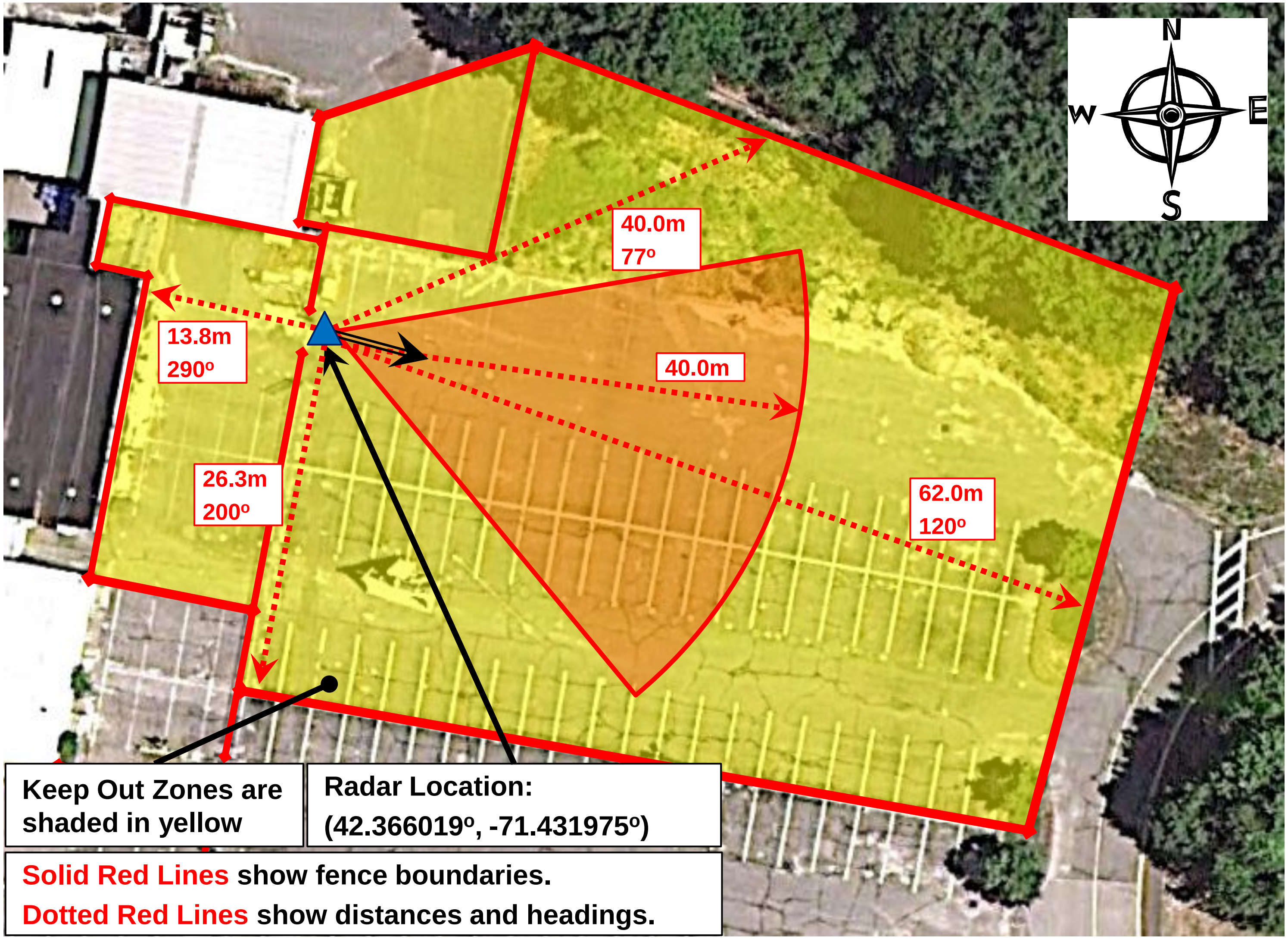

Keep Out Zones are shaded in yellow

Radar Location:
(42.366019°, -71.431975°)

Solid Red Lines show fence boundaries.
Dotted Red Lines show distances and headings.

Red shaded area shows azimuth coverage area and direction of radiation.

Raytheon Facility in Sudbury, MA
RF Exposure Limits Analysis Results

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- The following table lists the minimum safe distances for each Human Exposure limit
  - These are the minimum safe distances along the ground with the main lobe of the antenna pointed at the lower elevation limit for this site.

Requirement Description	Specification	Peak	Average	Analysis Results	
		mW/cm^2	mW/cm^2	Below Limit at X Distance (m)	Elevation Angle of Main Lobe (deg)
Human Exposure Limit					
C95.1 Controlled	IEEE C95.1-2005, Table 8		10.0	15	7
C95.1 Uncontrolled	IEEE C95.1-2005, Table 9		1.0	40	7
FCC, Controlled	47 CFR 1.1310		5.0	20	7
FCC, Uncontrolled	47 CFR 1.1310		1.0	40	7

Raytheon Facility in Pelham, NH
Diagram

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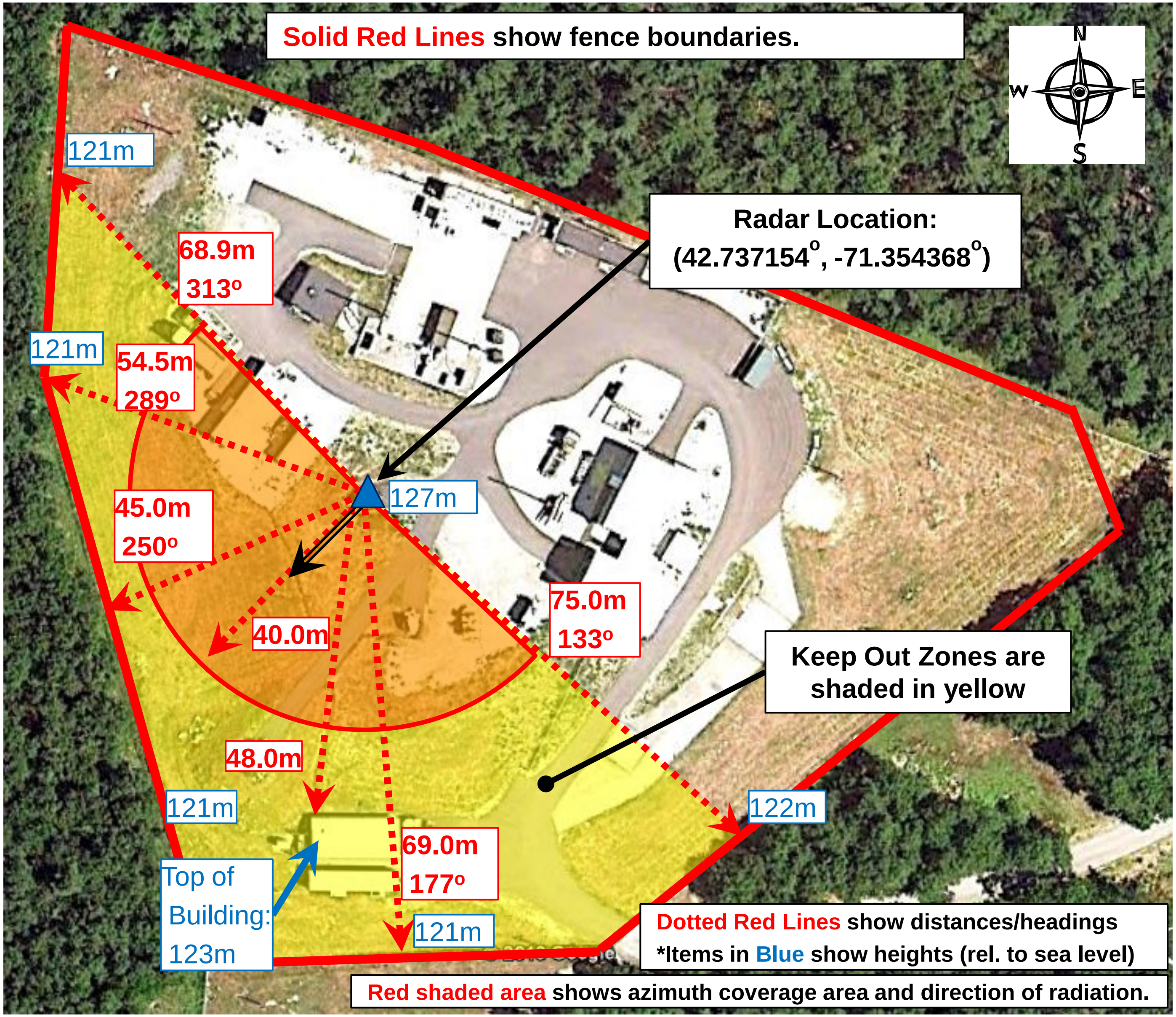

Raytheon Test Facility in Pelham, NH
Elevation Profile

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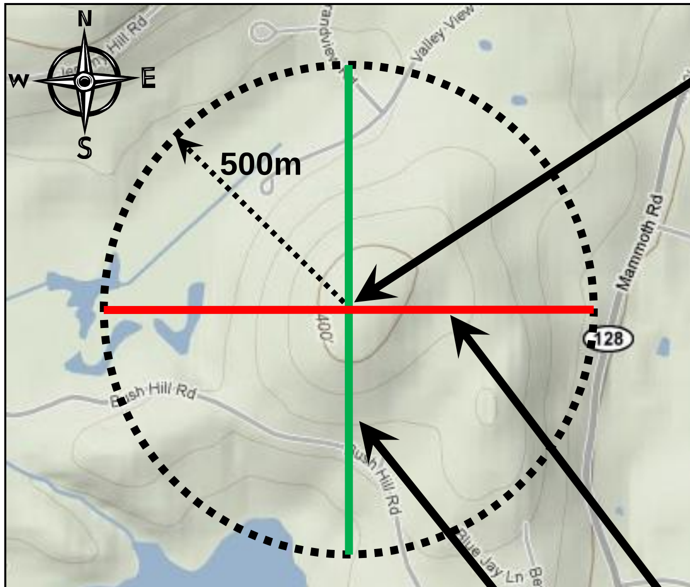
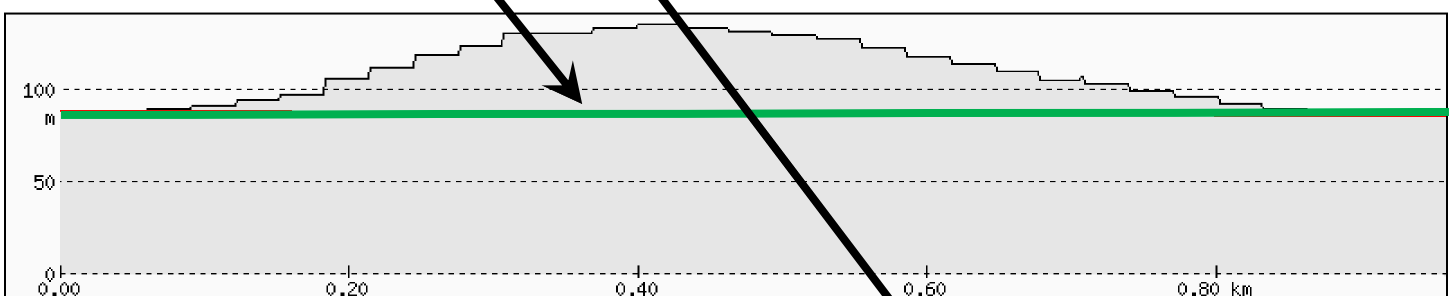
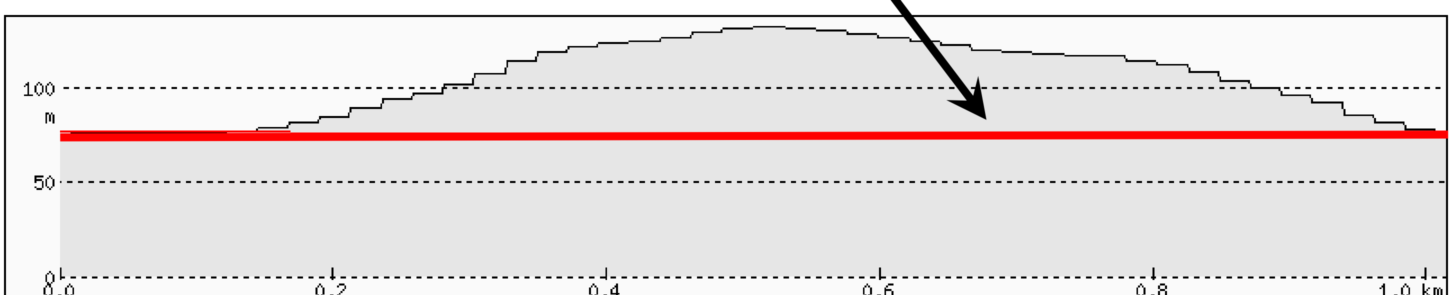


Radar Location:
(42.737154°, -71.354368°)

- Elevation profiles along each direction show that the Pelham, NH test site is located at the top of the hill.
  - Minimum safe distances on next slide are the distances along the ground going down the sides of the hill.





Raytheon Facility in Pelham, NH
RF Exposure Limits Analysis Results

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- The following table lists the minimum safe distances for each Human Exposure limit
  - These are the minimum safe distances along the ground with the main lobe of the antenna pointed at the lower elevation limit for this site.
  - From the radar location to the perimeter of the Keep Out Zones, the slope of the ground is approximately 8.5°

Requirement Description	Specification	Peak	Average	Analysis Results	
		mW/cm^2	mW/cm^2	Below Limit at X Distance (m)	Elevation Angle of Main Lobe (deg)
Human Exposure Level					
C95.1 Controlled	IEEE C95.1-2005, Table 8		10.0	9	0
C95.1 Uncontrolled	IEEE C95.1-2005, Table 9		1.0	40	0
FCC, Controlled	47 CFR 1.1310		5.0	20	0
FCC, Uncontrolled	47 CFR 1.1310		1.0	40	0

Analysis Results of Minimum Safe Distances For Applicable RF Exposure/Susceptibility Limits (1)

- The following slide summarizes the analysis results of the minimum safe distances for RF Exposure/Susceptibility Limits
  - – The results are detailed for each test facility

Analysis Results of Minimum Safe Distances For Applicable RF Exposure/Susceptibility Limits (2)

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Requirement Description	Peak	Average	Analysis Results			
			Sudbury, MA		Pelham, NH	
	mW/cm^2	mW/cm^2	Below Limit at X Distance (m)	Elevation Angle of Main Lobe (deg)	Below Limit at X Distance (m)	Elevation Angle of Main Lobe (deg)
Aircraft EMI/Susceptibility						
Military	10.6		255.9*	55	255.9*	55
Civilian	6631.4	28.9	48.5	55	48.5	55
Civilian VFR Rotorcraft	2387.3	23.9	53.5	55	53.5	55
Civilian	6631.4	28.9	48.5	55	48.5	55
Electro-Explosive Devices (in or on aircraft)	10.0		263.3*	55	263.3*	55
Human Exposure Limit						
C95.1 Controlled		10.0	15	7	9	0
C95.1 Uncontrolled		1.0	40	7	40	0
FCC, Controlled		5.0	20	7	20	0
FCC, Uncontrolled		1.0	40	7	40	0

*Note: These results were obtained using a Duty Factor of 100%, corresponding to the peak transmit power density.

The PRDS poses no RF hazard to personnel or aircraft.

Addendum and Backup

- The following slides show the power density calculation derivation in more detail and applicable references.

Power Density Equation
Derivation (1)

- The following slides will derive the power density equation in (1) used within this analysis
  - Established reference texts will be used to establish the foundation. See bibliography for list
- This derivation assumes:
  - Active phased array (same amplitude at each element)

Power Density Equation
Derivation (2)

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- The radiated field at any location from the i^th element in an array of N elements is given by[1]:

$$E_i(r,\theta,\phi) = f_i(\theta,\phi)\left\{\frac{1}{R_i}\left[a_i e^{-jkR_i}\right] + \frac{1}{R_i^2}[\ldots] + \ldots + \frac{1}{R_i^N}[\ldots]\right\} \qquad (5)$$

where

$$R_i = Distance\ from\ i^{th}\ element\ to\ observation\ point$$
$$f_i(\theta,\phi) = Element\ pattern\ gain$$
$$a_i = Output\ voltage\ amplitude\ at\ i^{th}\ element$$
$$k = Free\text{-}space\ wavenumber$$

- We can neglect the higher order $1/R_i^n$ terms in (5) to simplify this analysis. These higher order terms decay in the far-field of the antenna, defined as[3]:

$$R_{far-field} > \frac{2D^2}{\lambda} \qquad (6)$$

For our array dimensions, 1m x 1m, we can calculate the far-field region onset at an assumed frequency of 9.6GHz to be:

$$R_{far-field} > \left.\frac{2D^2}{\lambda}\right|_{\substack{D=1m \\ \lambda=3\cdot10^8/9.6\cdot10^9=0.0313\,m}} \quad \therefore R_{far-field} > 63.9m \approx 64m \qquad (7)$$

Power Density Equation Derivation (3)

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- With the higher order terms in (5) neglected, we can write the electric field at any point in the far-field as:

$$E_i(r,\theta,\phi) = f_i(\theta,\phi)\frac{a_i e^{-jkR_i}}{R_i} \qquad (8)$$

- We can write the total electric field due to all the elements in the array via the superposition principle. The total electric field is then written as[2]:

$$E_T(r,\theta,\phi) = \sum_{i=1}^{N} f_i(\theta,\phi)\frac{a_i e^{-jkR_i}}{R_i} \qquad (9)$$

- We now express the individual element weightings, a_i, as a complex weighting to steer the peak of the main-beam:

$$a_i = |a_i| e^{-jk r_i \cdot \hat{r}_0} \qquad (10)$$

where

$$\begin{aligned} r_i &= location\, of\ i^{th}\ element(x_i, y_i, z_i) \\ \hat{r}_0 &= \hat{x}u_0 + \hat{y}v_0 + \hat{z}\cos\theta_0 \\ u_0 &= \sin\theta_0 \cos\phi_0 \\ v_0 &= \sin\theta_0 \sin\phi_0 \end{aligned} \qquad (11)$$

Power Density Equation
Derivation (4)

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- We now express the electric field as

$$E_T(r,\theta,\phi) = \sum_{i=1}^{N} f_i(\theta,\phi)\left|a_i\right| e^{-jkr_i\cdot\hat{r}_0}\,\frac{a_i e^{-jkR_i}}{R_i} \qquad (12)$$

- We can assume that the amplitude, $|a_i|$, delivered to each element is equal, so we can bring that term out of the summation. We now assume that the gain per element is also equal, so that term can also be pulled out of the exponential.

$$E_T(r,\theta,\phi) = G_{TX,el}\left|a_{el}\right| \sum_{i=1}^{N} f_i(\theta,\phi) e^{-jkr_i\cdot\hat{r}_0}\,\frac{a_i e^{-jkR_i}}{R_i} \qquad (13)$$

- Since the power is proportional to the electric field via:

$$P \approx \left|E\right|^2 \qquad (14)$$

We can write the power at any point in the far-field due to our array of N elements as:

$$P_T(r,\theta,\phi) = \left| G_{TX,el}\left|a_{el}\right| \sum_{i=1}^{N} f_i(\theta,\phi) e^{-jkr_i\cdot\hat{r}_0}\,\frac{a_i e^{-jkR_i}}{R_i} \right|^2 \qquad (15)$$

Power Density Equation
Derivation (5)

- The element amplitude and gain can be pulled out of the absolute value. We recognize the term, $|a_{el}|^2$, as proportional to the output power per element in the array.

$$P_T(r,\theta,\phi) = G_{TX,el}{}^2 |a_{el}|^2 \left| \sum_{i=1}^{N} f_i(\theta,\phi) e^{-jkr_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i} \right|^2 \qquad (16)$$

- To put this in more relative terms, we can recognize that

$$G_{TX,el}{}^2 |a_{el}|^2 = \begin{cases} Element\,(Power)\,Gain = G_{TX,el}{}^2 \\ Peak\ Transmit\ Power\ Per\ Element = P_{TX} \propto |a_{el}|^2 \end{cases} \qquad (17)$$

In practice, the total antenna gain will include any element pattern gain as well. We can now write our power as:

$$P_T(r,\theta,\phi) = G_{TX,el}{}^2 P_{TX,el} \left| \sum_{i=1}^{N} f_i(\theta,\phi) e^{-jkr_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i} \right|^2 \qquad (18)$$

Power Density Equation Derivation (6)

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- We can now take the average power density per unit solid angle by recognizing that a solid angle can be defined as a steradian. A steradian is defined as[4] "the solid angle with its vertex at the center of a sphere of radius *r* that is subtended by a spherical surface area equal to that of a square with each side of length *r*". There are 4π steradians per sphere, so we can compute the average power density per unit solid angle by dividing (18) by 4π

$$P_T(r,\theta,\phi) = \frac{G_{TX,el}^{\;2} P_{TX,el}}{4\pi}\left|\sum_{i=1}^{N} f_i(\theta,\phi) e^{-jk r_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i}\right|^2 \qquad (19)$$

- For a pulsed radar system, we can replace $P_{TX,el}$ by $P_{TX,el} * DF$, where DF is the duty factor.

$$P_T(r,\theta,\phi) = \frac{G_{TX,el}^{\;2} P_{TX,el} DF}{4\pi}\left|\sum_{i=1}^{N} f_i(\theta,\phi) e^{-jk r_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i}\right|^2 \qquad (20)$$

- The element pattern, $f_i(\theta,\varphi)$, is often approximated as a cosine-function, defined by an element exponent, α.

$$f_i(\theta,\phi) \approx \sqrt{\cos^{\alpha}(\theta)} \qquad (21)$$

Power Density Equation
Derivation (7)

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- We can now combine the element gain and transmit power per element into a single unit known as the radiated power per element, $P_{rad,el}$. This includes the output power from the transmit channel, any front-end losses, and the element gain.

$$P_T(r,\theta,\phi) = \frac{P_{rad,el} DF}{4\pi} \left| \sum_{i=1}^{N} \sqrt{\cos^{\alpha}\theta} e^{-jk r_i \cdot \hat{r}_0} \frac{a_i e^{-jkR_i}}{R_i} \right|^2 \qquad (22)$$

- This completes the derivation of the power density equation. This formula aligns with the one derived in [6].

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